Physics 105

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Chapter -7-
(Linear momentum)

❖ Section (7.8): Center of Mass

- Center of Mass: It means the location of distribution of mass in space and it is ***the unique point*** at the center of a distribution of mass in space which has the property that the weighted position vectors relative to this point sum to zero.

- How we determine the position of center of mass?

I. System made up of two particles in ***one dimension***

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M} \quad \text{(Where } M = m_1 + m_2 \text{)}$$

II. System made up of more than two particles in ***one dimension***.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{M}$$

III. System particles in ***two dimensions***.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{M} \qquad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{M}$$

IV. Symmetrical extended objects:

✓ The position of CM is at the geometric center of the object.

- Determining the CM experimentally:

It is often easier to determine the CM or CG of an extended object experimentally rather than analytically. If an object is suspended from any point, it will swing due to the force of gravity on it, unless it is placed so its CG lies on a vertical line directly below the point from which it is suspended. If the object is two dimensional, or has a plane of symmetry, it need only be hung from two different pivot points and the respective vertical (plumb) lines drawn. Then the center of gravity will be at the intersection of the two lines, If the object doesn't have a plane of symmetry, the CG with respect to the third dimension is found by suspending the object from at least three points whose plumb lines do not lie in the same plane.

✓ ***Example:*** Find the position of the center of mass $m_1 = 6\ kg$, $m_2 = 2kg$, $x_1 = 2m$, $x_2 = 4m.$

✓ ***Solution:***

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{6 * 2 + 4 * 2}{2 + 6} = \frac{12 + 8}{8} = \frac{20}{8} = 2.5m$$

✓ ***Example***: Find the positions of the center of mass $m_1 = 1kg\ (4, 1)$, $m_2 = 5kg\ (3, 2)$, $m_3 = 7kg\ (-3, -5)$

✓ ***Solution:***

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{M}$$

$$= \frac{1 * (4) + 5 * (3) + 7 * (-3)}{1 + 5 + 7} = -0.153\ m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{M}$$

$$= \frac{1*(1)+5*(2)+7*(-5)}{1+5+7} = -1.8461\ m$$

❖ Section (7.9): CM for the human body

TABLE 7–1 Center of Mass of Parts of Typical Human Body, given as %
(full height and mass = 100 units)

Distance of Hinge Points from Floor (%)	Hinge Points (•) (Joints)	Center of Mass (×) (% Height Above Floor)		Percent Mass
91.2%	Base of skull on spine	Head	93.5%	6.9%
81.2%	Shoulder joint	Trunk and neck	71.1%	46.1%
	elbow 62.2%‡	Upper arms	71.7%	6.6%
	wrist 46.2%‡	Lower arms	55.3%	4.2%
52.1%	Hip joint	Hands	43.1%	1.7%
		Upper legs (thighs)	42.5%	21.5%
28.5%	Knee joint			
		Lower legs	18.2%	9.6%
4.0%	Ankle joint	Feet	1.8%	3.4%
		Body CM =	58.0%	100.0%

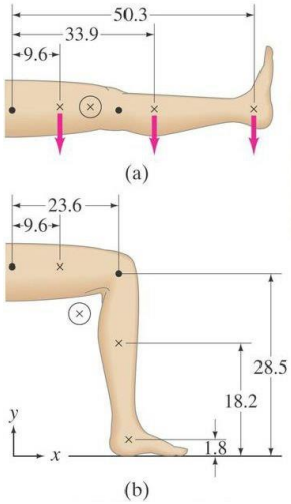
‡ For arm hanging vertically.

✓ ***Example:*** Determine the center of mass of a whole leg ($m_1 = 21.5$, $m_2 = 9.6$, $m_3 = 3.4$)

I. When stretched out.

II. When bent at 90°.

(*Assume the person is 1.70 m tall*)



✓ ***Solution:***

I. $x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$

$$x_{cm} = \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4}$$

$= 20.4$ Units From the hip Joint

(from table) 52.1 – 20.4 = 31.7 units (From the base to the foot)

To express the result in meters we need to multiplied it by ($\frac{1.7m}{100}$)

((1.7 is the person tall))

$$x_{cm} = \frac{1.7 * 31.7}{100} = 0.54\ m$$ ((above the bottom of the foot))

II. We use an x-y coordinate system, as shown b. First, we calculate

How far to the right of the hip joint the CM lies, accounting for all three parts:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M}$$

$$x_{cm} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{21.5 + 9.6 + 3.4}$$

$x_{cm} = 14.9\ units,, 0.25m$ $(from\ the\ hip\ joint)$.

we calculate the distance, of the CM above the floor:

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M}$$

$$y_{cm} = \frac{((3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{3.4 + 9.6 + 21.5}$$

$y_{cm} = 23.0\ units,, 0.39m$ $(from\ the\ hip\ joint)$

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